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Testing the Performance of Symmetric and Asymmetric GARCH Models in Nigeria's Crude Oil Markets between 1987 and 2023

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Abstract: This paper analyses Nigerian crude oil markets from 1987 to 2023 using symmetric and asymmetric GARCH models. The specific goals include examining the ARCH effect in Nigeria's monthly crude oil price returns from January 1987 to April 2023, assessing the impact of volatility on these returns using the symmetric GARCH model, investigating the impact of leverage effects and news using the asymmetric model, and comparing the performance of both models. The Central Bank of Nigeria (CBN) Statistical Database provided crude oil prices in Naira/Dollar from January 1999 to April 2023. Since the raw series and Nigerian crude oil price returns show a steady trend over time, the model estimation findings reveal a non-stationary price series. The results indicate that both symmetric and asymmetric GARCH models can effectively model Nigerian crude oil returns. The Akaike information criteria showed that the EGARCH model under the Student-t distribution with fixed degrees of freedom provided the best fit. Diagnostic tests reveal that price shocks can have lasting consequences, supporting the EGARCH model's analysis of Nigerian crude oil prices. The long memory hypothesis shows that the Nigerian crude oil market is unstable and evolving. The study made many recommendations based on these findings.

Keywords: Crude Oil Prices; Symmetric GARCH; Asymmetric GARCH; ARCH Effect; Leverage Effect; Student-t Distribution; Nigerian Crude Oil Market; Inflation Rates; International Trade.

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1. Introduction

Financial time series, including stock prices, exchange rates, inflation rates, and crude oil prices, often exhibit patterns of aggregation and volatility clustering. Typically, periods of dramatic price changes are followed by phases of relative stability, a trend also evident in macroeconomic factors such as employment, consumption, investment, and international trade. This ongoing volatility plays a significant role in causing global economic shocks, particularly affecting developing nations through fluctuations in trade and prices. Crude oil is one of the most crucial and limited global resources. Even with the rising interest in renewable energy, it still holds a dominant position in international markets. The history of its pricing shows extreme volatility influenced by financial crises, supply shortages, and geopolitical conflicts. For instance, the Asian financial crisis in 1999 and the housing and credit crisis in the U.S. in 2008 resulted in sharp declines in oil prices. Conversely, limited supply and tense relations between the U.S. and oil-producing nations led to prices soaring to unprecedented heights of $147.30 per barrel [7]. Like other commodities, the price of oil swings between scarcity and abundance, creating ripple effects across

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multiple sectors, including transport fuels and consumer goods [2]. The fluctuations in crude oil prices have significant consequences for production expenses, consumer pricing, and national energy strategies.

When oil prices rise, companies face higher production costs, which are typically passed on to consumers; this also affects government energy policies and subsidies [4]. Therefore, reliable forecasts of crude oil prices are essential for investors, researchers, and policymakers. Current research indicates that Generalised Autoregressive Conditional Heteroskedasticity (GARCH) models and their variations are commonly employed to understand volatility features, including clustering, persistence, heavy tails, and leverage effects [16]; [11]. However, the significance of error distribution assumptions has been insufficiently addressed, despite their strong impact on the accuracy of volatility modelling [14]. This research aims to analyse and predict the volatility of Nigerian monthly crude oil price data from 1987 to 2023, employing both symmetric (GARCH and GARCH-M) and asymmetric (EGARCH, TGARCH, GJRGARCH, CGARCH, and APARCH) models based on three error distribution assumptions: normal, Student-t, and generalised error distribution. The objective is to compare the performance of these models and determine which framework is best suited for capturing the dynamics of crude oil prices in Nigeria [8].

2. Methodology

The presence of conditional heteroskedasticity, if not accounted for, leads to misleading results. Autoregressive conditional heteroscedasticity (ARCH) is employed to assess the prevalence of heteroscedasticity in the residuals of the return series using the Lagrange Multiplier (LM) test. Engle [9] proposed the ARCH model (Autoregressive Conditional Heteroskedasticity). Every ARCH or GARCH family model requires two distinct specifications: the mean equation and the variance equation. According to Engel, conditional heteroskedasticity in a return series, y_t can be modelled using the ARCH model, expressing the mean equation in the form:

$$\mathbf{y_t = E_{t-1}(y_t) + \varepsilon_t}$$

Where, $\boldsymbol{\varepsilon_t = \varphi_t \sigma_t}$

$\boldsymbol{\varepsilon_t}$ is the error gotten from the mean equation at time t.

$\mathbf{E_{t-1}}$ is the expectation conditioned on the information available at time t-1.

$\boldsymbol{\varphi_t}$ is a sequence of independent, identically distributed random variables whose mean is zero with unit variance. The variance equation for an ARCH model of order q is given as:

$$\boldsymbol{\sigma_t}^2 = \boldsymbol{\alpha_0} + \sum_{i=1}^{q} \boldsymbol{\alpha_i \varepsilon_{t-i}^2} + \boldsymbol{\mu_t}$$

$$= \boldsymbol{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + +\alpha_2 \varepsilon_{t-2}^2 + \alpha_3 \varepsilon_{t-3}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2}$$

Where, $\boldsymbol{\alpha_0 > 0}$, $\boldsymbol{\alpha_i \geq 0}$, $\boldsymbol{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots \varepsilon_{t-p}}$ are independent for all t and $\mathbf{i = 1, 2, 3, \ldots, q}$ is the number of lags.

In the practical application of the ARCH (q) model, the decay rate is usually more rapid than what actually applies to financial time series data. To account for this, the order of the ARCH must be at a maximum, a process that is strenuous and more cumbersome. In another development, some drawbacks of the ARCH model include the requirement to estimate the coefficients of p autoregressive terms, which can consume several degrees of freedom. It is often difficult to interpret all the coefficients, especially if some of them are negative. The OLS estimating procedure does not lend itself to estimating the mean and variance functions simultaneously. Therefore, the literature suggests that an ARCH model higher than ARCH (3) is better estimated by the Generalised Autoregressive Conditional Heteroscedasticity (GARCH) model. Thus, the GARCH model was introduced. Bollerslev [10] proposed the generalised autoregressive conditional heteroskedasticity (GARCH) model as an alternative to the ARCH model. This model differs from the ARCH model in that it incorporates squared conditional variance terms as additional explanatory variables. This allows the conditional variance to follow an ARMA process. If we write the residual as:

$$\boldsymbol{\sigma_t}^2 = \boldsymbol{\beta_0} + \sum_{i=1}^{q} \boldsymbol{\alpha_i \varepsilon_{t-i}^2} + \sum_{i=1}^{p} \boldsymbol{\beta_j \varepsilon_{t-j}^2}$$

Where p is the order of the GARCH terms, σ^2 and q is the order of the ARCH terms, $\boldsymbol{\varepsilon^2}$.

Where, $\beta_0 > 0, \alpha_i > 0, i = 1, 2, 3 \ldots q - 1,\ \ j = 1, 2, 3, \ldots, p - 1$, $\sigma_t^{\ 2}$ is the conditional variance and $\varepsilon^2{}_t$, disturbance term. The reduced form of equation 3 is the GARCH (1, 1) represented as:

$$\sigma^{\ 2}_{\ t} = \beta_0 + \beta_1 \varepsilon^2_{t-1} + \beta_2 \sigma^2_{t-1}$$

$$R = \mu + \varepsilon_t$$

Where, $\varepsilon_t^2 \sim N(0,\ \sigma^{\ 2}_{\ t})$

- μ is the mean of the return.
- $\sigma^{\ 2}_{\ t}$ is the error variance at time t.
- ε^2_{t-1} is the squared error at time t-1.

The three parameters (β_0, $\beta_1$and β_2) are nonnegative and β_1+ β_2 <1 to achieve stationarity. The limitation of the model, the principal restriction of this model, is that all the explanatory variables in a GARCH and therefore ARCH model must be positive; this is known as the non-negativity constraint. Clearly, it is impossible to have a negative variance, as it consists of squared variables. Additionally, the primary issue with an ARCH model is that it necessitates a substantial number of lags to capture the nature of volatility, which can be problematic as it is challenging to determine the optimal number of lags to include, resulting in a non-parsimonious model where the non-negativity constraint may be violated. The GARCH model is typically more parsimonious, and often a GARCH(1,1) model is sufficient. This is because the GARCH model incorporates much of the information that a much larger ARCH model with large numbers of lags would contain.

3. GARCH-in-Mean

In this class of models, the conditional variance is included in both the conditional mean equation and the usual error variance part.

$$y_t = \mu + \delta \sigma_{t-1} + u_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u^2_{t-1} + \beta \sigma^2_{t-1}$$

If y_t is assumed to be an asset return, then in effect the first equation suggests that the mean return is dependent on the risk. If the parameter δ is positive and significant, it means that the mean return increases with greater risk. In effect, δ can be interpreted as a risk premium. The shortcoming of symmetric GARCH models is that the conditional variance does not react asymmetrically to fluctuations in returns. As a result, several models, known as asymmetric models, such as EGARCH and TGARCH, among others, have been developed to address this issue. In line with Banumathy and Azhagaiah [5], the relationship between asymmetric volatility and stock returns was examined using the Exponential GARCH (EGARCH) and Threshold GARCH (TGARCH) models. The EGARCH model is hinged on a logarithmic expression of the conditional variability. The EGARCH model allows for the testing of the presence of the leverage effect. The presence of the leverage effect or asymmetry is tested based on the hypothesis that $\gamma_i < 0$, the impact is said to be symmetric if $\gamma_i \neq 0$.

3.1. Exponential GARCH (EGARCH) Model

The conditional variance of the EGARCH (p, q) model is specified generally as:

$$\log(\sigma^{\ 2}_{\ t}) = \beta_0 + \sum_{i=1}^{q} \left\{\alpha_1 \left|\frac{\varepsilon_{\ t-i}}{\sigma_{t-i}}\right| + \gamma\left(\frac{\varepsilon_{\ t-i}}{\sigma_{t-i}}\right)\right\} + \sum_{j=1}^{p} \beta_j \log\left(\sigma^{\ 2}_{\ t-j}\right)$$

$\varepsilon_{\ t-i} > 0$ implies good news while $\varepsilon_{\ t-i} < 0$ implies bad news. Their total effects are $(1 + \gamma_i)|\varepsilon_{\ t-1}|$ and $(1 - \gamma_i)|\varepsilon_{\ t-1}|$, respectively. When $\gamma_i < 0$, the expectation is that bad news would have a higher impact on volatility. The EGARCH model achieves covariance stationarity when $\sum_{j=1}^{p} \beta_j < 1$. The interest of this research is to model the conditional variance using the GARCH models earlier specified and the EGARCH (1,1) model, which is specified as:

$$\log(\sigma^{\ 2}_{\ t}) = \beta_0 + \alpha_1 \left|\frac{\varepsilon_{\ t-1}}{\sigma_{t-1}}\right| + \gamma\left(\frac{\varepsilon_{\ t-1}}{\sigma_{t-1}}\right) + \beta_j \log\left(\sigma^{\ 2}_{\ t-j}\right)$$

The total effects of good news and bad news for EGARCH (1,1) are $\boldsymbol{\gamma_1)|\varepsilon_{t-1}|}$ and $\boldsymbol{(1-\gamma_1)|\varepsilon_{t-1}|}$ Respectively. Failing to accept the null hypothesis that $\boldsymbol{\gamma_1 = 0}$ indicates the presence of the leverage effect, which states that bad news has a stronger impact than good news on the volatility of stock index returns. The EGARCH model has several advantages over the basic GARCH model, one of which is that the non-negativity constraint does not need to be imposed on the system. Although the EGARCH models have their own shortcomings, the principle is the restriction of the model. In this model, all explanatory variables are included in the GARCH component; therefore, the ARCH component must be nonnegative, which is known as the non-negativity constraint. Clearly, it is impossible to have a negative variance, as it consists of squared variables.

3.2. GJR- GARCH Model

The GARCH model can be extended to include any number of lags on the squared error term and the conditional variance term. The GARCH (p,q) model has p lags on the conditional variance term and q on the squared error term. However, in general, a GARCH (1,1) model is sufficient. Asymmetric GARCH models are attributed to the leverage effect in asset prices, where a positive shock has a less pronounced effect on the conditional variance compared to a negative shock. This can be incorporated into the GARCH model using a dummy variable. This was introduced by Glosten et al. [6] (GJR) and demonstrated that asymmetric adjustment was a crucial consideration in asset prices. The model is of the form:

$$\boldsymbol{\sigma_t^2 = \beta_0 + \alpha_1\varepsilon_{t-1}^2 + \tau_1 I_{t-1}\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2}$$

Where I is a dummy variable that takes the value of 1 when the shock is less than 0 (negative) and zero otherwise, to determine if there is asymmetric adjustment, which depends on the significance of the last term, which can be determined using the t-statistics. In this model, good news implies that $\boldsymbol{\varepsilon_{t-i}^2 > 0}$ and bad news implies that $\boldsymbol{\varepsilon_{t-i}^2 < 0}$ and these two shocks of equal size have differential effects on the conditional variance. Good news has an impact on $\boldsymbol{\alpha_i}$ and bad news has an impact on $\boldsymbol{\alpha_i + \tau_j}$. Bad news increases volatility when$\boldsymbol{\tau_j > 0}$, which implies the existence of the leverage effect in the i-th order, and when $\boldsymbol{\tau_j \neq 0}$ The news impact is asymmetric. Asymmetric GARCH models are attributed to the leverage effect in asset prices, where a positive shock has a less pronounced effect on the conditional variance compared to a negative shock. This can be incorporated into the GARCH model using a dummy variable.

3.3. The Power GARCH (PGARCH) Model

Ding et al. [19] expressed conditional variance using PGARCH (p, d, q) as:

$$\boldsymbol{\sigma_t^d = \beta_0 \sum_{i=1}^{p} \alpha_i\left(|\varepsilon_{t-i}| + \gamma_i\varepsilon_{t-i}\right)^d + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^d}$$

Here, $\boldsymbol{d > 0}$ and $\mathbb{R}^+$, $\boldsymbol{\gamma_i < 1}$ establishes the existence of leverage effects. If d is set at 2, the PGARCH (p, q) replicates a GARCH (p, q) with a leverage effect. If d is set at 1, the standard deviation is modelled. The first order of equation 3.10 is PGARCH (1, d, 1), expressed as:

$$\boldsymbol{\sigma_t^d = \beta_0 + \alpha_1(|\varepsilon_{t-1}| + \gamma_1\varepsilon_{t-1})^d + \beta_1\sigma_{t-1}^d}$$

The failure to accept the null hypothesis that $\boldsymbol{\gamma_1 \neq 0}$ shows the presence of the leverage effect. The impact of news on volatility in PGARCH is like that of TGARCH when $\boldsymbol{d}$ is 1.

3.4. Component GARCH Model

According to Zhang et al. [12], the Component GARCH model was proposed by Engle [9]. It constitutes a convenient method of incorporating long-memory-like features into a short-memory model, at least for the horizons relevant for option valuation. The model decomposes conditional variance into a long-term transitory component. The equation of the C-GARCH model is:

$$\boldsymbol{\sigma_t^2 = q_t + \sum_{i=1}^{m} \alpha_i\left(r_{t-1}^2 - q_{t-1}\right) + \sum_{j=1}^{s} \beta_j\left(\sigma_{t-j}^2 - q_{t-j}\right)}$$

$$\boldsymbol{q_t = \alpha_0 + Pq_{t-1} + \varphi\left(r_{t-1}^2 - \sigma_{t-1}^2\right)}$$

$\boldsymbol{q_t}$, $\boldsymbol{\alpha_i}$, $\boldsymbol{\beta_j}$, $\boldsymbol{\varphi}$, and p are unknown parameters, and $\boldsymbol{\alpha_0}$ is the intercept term. In this model, the intercept term $\boldsymbol{q_t}$ is regarded as a time-varying process, which represents the long-term component of the conditional variance. The difference between the conditional variance and the long-term component $\boldsymbol{\left(\sigma_{t-j}^2 - q_{t-j}\right)}$ is the transitory component that models short-term volatilities.

The Component GARCH model accounts for both the long-term and short-term variations in the data series. It applies in situations when there is an obvious trend in the data.

3.5. Error Distributional Assumptions

In modelling the conditional variance of crude oil price, five conditional distributions for the standardised residuals of the price returns modernism were considered, and they include the Gaussian (Normal), Student's t, Generalised, Student's t with fixed parameter(v=3), and Generalised with fixed parameter(v=3).

3.6. The Gaussian (Normal) Distribution

The Gaussian (Normal) error distribution assumed a log-likelihood contribution of the form.

$$\mathbf{Log\,L\,(\theta_t) = \sum_{t=1}^{T} L(\theta_t) = -\frac{1}{2}\sum_{t=1}^{T} \log\,(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{(y_t - \gamma y_t)^2}{\sigma_t^2}}$$

3.7. The Student's s-t Error Distribution

The student's t error distribution assumes a log-likelihood contribution is of the form;

$$\mathbf{Log\,L\,(\theta_t) = -\frac{1}{2}\log\left[\frac{\prod[\nu-2]\Gamma\left[\frac{\nu}{2}\right]^2}{\Gamma\left[\frac{\nu+1}{2}\right]^2}\right] - \frac{1}{2}Log\,\sigma_t^2 - \frac{[\nu+1]}{2}Log\left[1 + \frac{[y_{t-}X_t'\gamma]^2}{\sigma_t^2\Gamma[\nu-2]}\right]}$$

Where $\boldsymbol{\sigma_t^2}$ is the variance at time t, and the degree of freedom $\boldsymbol{\nu > 2}$ controls the tail behaviour.

3.8. The Generalised Error Distribution

The Generalised Error Distribution (GED) likelihood function is specified as thus:

$$\mathbf{Log}\,\boldsymbol{L\,(\theta_t)} = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{log}\left[\frac{\boldsymbol{\Gamma}\left[\frac{\mathbf{1}}{\boldsymbol{r}}\right]^3}{\boldsymbol{\Gamma}\left[\frac{\mathbf{3}}{\boldsymbol{r}}\right]\left[\frac{\boldsymbol{r}}{\mathbf{2}}\right]^2}\right] - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{Log}\,\boldsymbol{\sigma_t^2} - \mathbf{Log}\left[\frac{\boldsymbol{r}\frac{\mathbf{3}}{\boldsymbol{r}}[\boldsymbol{y_{t-}X_t'\gamma}]^2}{\boldsymbol{\sigma_t^2}\boldsymbol{\Gamma}\left[\frac{\mathbf{1}}{\boldsymbol{r}}\right]}\right]^{\frac{r}{2}}$$

$\mathbf{r > 0}$ is the shape of the parameter, which basically accounts for the skewed properties of the returns of the series under estimation. The higher the value of r, the heavier the weight of the tail. Deebom and Essi [14] revealed that the Generalised (GED) distribution turns out to be a Gaussian (Normal) error distribution if r = 0 and flat-tailed if r < $\mathbf{2}$.

3.9. The student's-t with Fixed Parameter (v = 3) Error Distribution

The student's t error distribution assumes a log-likelihood contribution is of the form;

When v = 3, we substitute the value into the model equation.

$$\mathbf{Log}\,\boldsymbol{L\,(\theta_t)} = -\frac{\mathbf{1}}{\mathbf{2}}\mathbf{log}\left[\frac{\prod\boldsymbol{\Gamma}\left[\frac{\mathbf{3}}{\mathbf{2}}\right]^2}{\boldsymbol{\Gamma}[\mathbf{2}]^2}\right] - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{Log}\,\boldsymbol{\sigma_t^2} - \mathbf{2Log}\left[\mathbf{1} + \frac{[\boldsymbol{y_{t-}X_t'\gamma}]^2}{\boldsymbol{\sigma_t^2}}\right]^2$$

Where $\boldsymbol{\sigma_t^2}$ is the variance at time t, and the degree of freedom $\boldsymbol{\nu = 3}$ controls the tail behaviour.

3.10. The Generalised with Fixed Parameter (v=3) Error Distribution

The Generalised (GED) likelihood function is specified as thus:

When v=3, we substitute the value into the model in equation (3.24), we have

$$\mathbf{Log\,L\,(\theta_t) = -\frac{1}{2}\log\left[\frac{\Gamma\left[\frac{1}{r}\right]^3}{\Gamma\left[\frac{3}{r}\right]\left[\frac{r}{2}\right]^2}\right] - \frac{1}{2}Log\,\sigma_t^2 - Log\left[\frac{r\frac{3}{r}[y_{t-}X_t'\gamma]^2}{\sigma_t^2\Gamma\left[\frac{1}{r}\right]}\right]^{\frac{r}{2}}}$$

$$\mathbf{Log\,L\,(\theta_t) = -\frac{1}{2}\log\left[\frac{\Gamma\left[\frac{1}{3}\right]^3}{\Gamma\left[\frac{3}{3}\right]\left[\frac{3}{2}\right]^2}\right] - \frac{1}{2}Log\,\sigma_t^2 - Log\left[\frac{3\frac{3}{3}[y_{t-}X_t'\gamma]^2}{\sigma_t^2\Gamma\left[\frac{1}{3}\right]}\right]^{\frac{3}{2}}}$$

$$\mathbf{Log\,L\,(\theta_t) = -\frac{1}{2}\log\left[\frac{\Gamma\left[\frac{1}{9}\right]}{\Gamma\left[\frac{9}{4}\right]}\right] - \frac{1}{2}Log\,\sigma_t^2 - Log\left[\frac{3[y_{t-}X_t'\gamma]^2}{\sigma_t^2\Gamma\left[\frac{1}{r}\right]}\right]^{\frac{3}{2}}}$$

v=3 is the shape of the parameter, which basically accounts for the skewed properties of the returns of the series under estimation. The higher the value of v, the heavier the weight of the tail. The data for the study is Nigerian crude oil price (Naira/Dollar), which was sourced from and extracted from the Central Bank of Nigeria (CBN) statistical database website (www.cbn.gov.ng). The data spanned from January 1982 to January 2023. Returns on Price in Nigeria's crude oil markets are estimated using the following equation:

$$\mathbf{RCOPt \quad = \quad Log\left(\frac{COPt}{COP_{t-1}}\right) \times \frac{100}{1}}$$

For $\mathbf{t = 1, 2 \ldots\ldots t - j}$ where $\mathbf{COPt}$ is the Crude oil price at time t, and $\mathbf{COPt - 1}$ is the Crude oil price at time " $\mathbf{t} - \boldsymbol{j}$" the variable is well transformed to get rid of outliers and also obtain stationarity among them. The software used for data analysis was Econometric View (EViews).

4. Results

The results of the study are presented as follows: Figure 1 is a time plot of the crude oil price from January 1987 to September 2022, while Figure 2 represents a time plot of the returns on the crude oil price from January 1987 to September 2022.

Figure 1: Time plot of crude oil price from January 1987 to September 2022

On the horizontal axis, we have the raw data, while on the vertical axis is time (years). Figure 2 illustrates the dynamics of the crude oil price series.

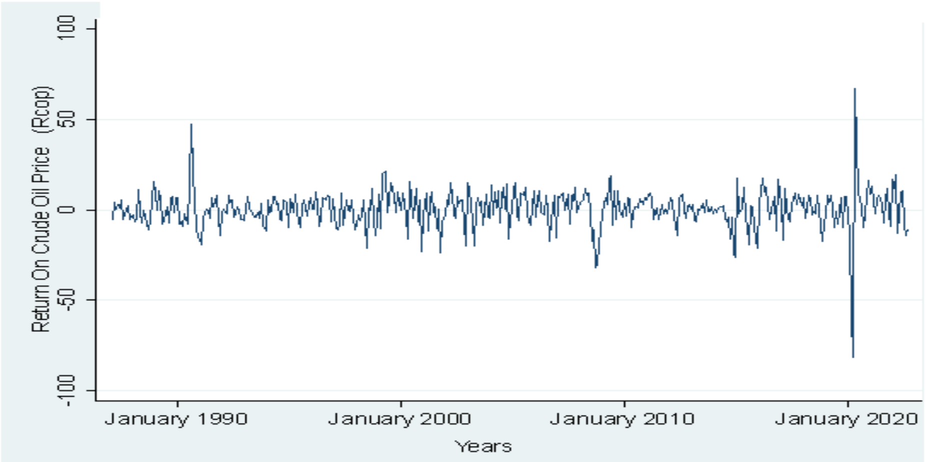

Figure 2: Time plot of the returns on crude oil price from January 1987 to September 2022

From a visual examination of the behaviour of crude oil prices from January 2006 to June 2008, a rise was followed by a fall in 2008. This reveals that the crude oil price fluctuates across the period under investigation, necessitating the need to detrend the series. The time plot illustrates the direction of the trend in the variables (crude oil price) under investigation. The time plot of the raw data on Nigeria's crude oil Price (Naira/Dollar) is consistent with Deebom et al. [17] and Deebom et al. [18] 's results on comparative modeling of price volatility in Nigerian Crude Oil Markets Using Symmetric and Asymmetric GARCH Models [18]. However, the only difference is that the data spanned from January 1982 to December 2023.

Table 1: Descriptive statistics for crude oil price and the returns on crude oil price

Variable	Mean	Median	Maxi	Min	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	P-value
COP	49.043	37.78	138.74	10.22	33.718	0.796	2.435	50.924	0.000
RCOP	0.382	0.679	66.977	-81.590	10.781	-0.855	16.092	3108.906	0.000

The descriptive statistics are presented in Table 1 for the crude oil price and the returns on the crude oil price. The emphasis of these results is on central moments, Standard deviation, skewness, and kurtosis. The mean for crude oil price (49.043) and the returns on crude oil price (0.382) are positive, while the standard deviations (33.718) for crude oil price and (10.781) for the returns on crude oil price, respectively. However, it is essential to note that the mean statistic may not capture all the information in a time series, particularly when significant variations, trends, or seasonality are present. Therefore, it is essential to consider other statistical measures and methods to gain a comprehensive understanding of the data. Additionally, the skewness test, as presented in Table 1, yields a positive result (0.796) for the crude oil price, while the returns on the crude oil price are negative (-0.855) respectively. The negative skewness in time series estimates indicates a departure from symmetry, suggesting that caution should be exercised when applying certain statistical analyses or forecasting techniques (Table 2).

Table 2: Engle's ARCH-effect and Ljung-Box-Pierce Q-test on returns on crude oil price

Lagged	ARCH		Ljung-Box-Pierce Q-test	
	F-statistic	$n * R^2$	F-statistic	$n * R^2$
Q (1)	219.743 Prob. F (1,424) (0.000)	145.416 $x^2(1)$ (0.000)	8.767 Prob. F (1,424) (0.003)	8.651 $x^2(1)$ (0.003)
Q (5)	50.173 Prob. F (5,416) (0.000)	158.750 $x^2(5)$ (0.0000)	2.879 Prob. F (5,420) (0.001)	14.151 $x^2(5)$ (0.001)
Q (10)	24.725 Prob. F (10,406) (0.000)	157.831 $x^2(10)$ (0.000)	1.707 Prob. F (10,415) (0.077)	16.874 $x^2(5)$ (0.077)

Understanding the implications of negative skewness helps in making informed decisions and choosing appropriate methodologies for data analysis and modelling. Additionally, the data series on crude oil price kurtosis (2.435) exhibits a lower kurtosis, whereas the returns on the crude oil price series have a higher kurtosis (16.092). The high kurtosis exhibits the attributes of a price return series. A time series estimate with high kurtosis statistics indicates heavy tails, non-normality, and an increased risk, highlighting the need for appropriate modelling techniques and statistical methods that account for heavy-tailed distributions (Table 3).

Table 3: Results of the symmetric GARCH model for returns on crude oil price (RCOP)

Model	Parameter	NED	STD	GED (1.474)	STDt (df = 10) ED	GED (df =1.5)	MIN AIC
ARCH (1)	C	44.921(0.000)	46.357(0.000)	45.235(0.000)	45.261(0.000)	45.110(0.000)	
	$\varepsilon^2_{1,t-1}$	0.569(0.000)	0.523(0.000)	0.551(0.000)	0.509(0.000)	0.550(0.000)	
	AIC	7.173	7.138	7.149	7.135	7.145	ARCH (1) STD
ARCH (2)	C	54.676(0.000)	54.446(0.000)	54.855(0.000)	53.632(0.000)	55.110(0.000)	
	ε^2_{1t-1}	0.483(0.000)	0.467(0.001)	0.476(0.000)	0.461(0.000)	0.478(0.000)	
	ε^2_{2t-1}	-0.133(0.134)	-0.123(0.273)	-0.134(0.207)	-0.12290.267)	-0.133(0.217)	
	AIC						
GARCH (1,1)	C	2.713(0.090)	3.285(0.000)	0.057(0.820)	0.083(0.746)	0.065(0.796)	
	ε_{t-1}	0.620(0.000)	0.489(0.000)	0.185(0.000)	0.179(0.000)	0.187(0.000)	
	C	0.533(0.000)	0.591(0.000)	3.260(0.073)	3.187(0.054)	3.259(0.080)	
	ε^2_{t-1}	2.713(0.000)	3.285(0.000)	0.529(0.000)	0.475(0.000)	0.525(0.000)	
	σ^2_{t-1}	0.620(0.000)	0.489(0.00)	0.570(0.000)	0.595(0.000)	0.574(0.000)	
	AIC	7.133	7.104	7.092	7.073	7.088	
VOLATILITY	$c(4)\varepsilon^2_{t-1} + c(5)\sigma^2_{t-1}$	3.333	3.774	1.099	1.07	1.099	
	@SQRT(GARCH)	0.165(0.100)	0.190(0.056)	0.172(0.076)	0.190(0.061)	0.172(0.071)	
	C	-0.944(0.123)	-1.000(0.117)	-0.916(0.131)	-1.001(0.119)	-0.904(0.132)	
	ε_{t-1}	0.175(0.000)	0.172(0.000)	0.181(0.000)	0.172(0.000)	0.182(0.000)	
GARCH-M	C	2.542(0.080)	2.589(0.095)	2.474(0.122)	2.598(0.000)	2.458(0.000)	
	ε^2_{t-1}	0.586(0.000)	0.468(0.000)	0.523(0.000)	0.471(0.000)	0.518(0.000)	
	σ^2_{t-1}	0.554(0.000)	0.615(0.000)	0.587(0.000)	0.608(0.000)	0.592(0.000)	
	AIC	7.101	7.070	7.086	7.066	7.082	
VOLATILITY	$c(4)\varepsilon^2_{t-1} + c(5)\sigma^2_{t-1}$	1.14	1.083	1.11	1.079	1.11	

To test the null hypothesis, which states that the data is normally distributed, against the alternative hypothesis that it is not normally distributed. The results show that the test p-value is less than the predefined significance level of 5%. Therefore, we can reject the null hypothesis and accept the alternative hypothesis that the series are not normally distributed (Table 4).

Table 4: Results of the asymmetric GARCH model for returns on crude oil price (RCOP)

Model	Parameter	NED	STD	GED (1.474)	STDt (df = 10) ED	GED (df =1.5)	MIN AIC
EGARCH (1,1)	C	-0.241(0.411)	-0.098(0.738)	-0.129(0.660)	-0.098(0.738)	-0.103(0.7196)	
	ε_{t-1}	0.222(0.000)	0.210(0.000)	0.2178(0.000)	0.210(0.000)	0.220(0.0000)	
	C	0.014(0.929)	-0.015(0.919)	-0.002(0.988)	-0.015(0.919)	-0.008(0.9644)	
	ε^2_{t-1}	0.707(0.000)	0.605(0.000)	0.659(0.000)	0.605(0.000)	0.645(0.000)	
	I^2_{t-1}	-0.126(0.003)	-0.115(0.059)	-0.115(0.000)	-0.115(0.059)	-0.114(0.056)	
	σ^2_{t-1}	0.866(0.000)	0.891(0.000)	0.8786(0.000)	0.891(0.000)	0.883(0.000)	
	AIC	7.0634	7.041	7.056	7.037	7.054	

VOLATILITY	$\mathbf{c(4)\varepsilon_{t-1}^2 + c(5)\sigma_{t-1}^2}$	1.573	1.496	1.5376	1.496	1.528	
	C	-0.234(0.416)	-0.060(0.834)	-0.103(0.721)	-0.074(0.800)	-0.083(0.768)	
	$\boldsymbol{\varepsilon_{t-1}}$	0.195(0.000)	0.196(0.000)	0.200(0.000)	0.196(0.000)	0.202(0.000)	
GJR-GARCH	C	3.928(0.026)	3.237(0.050)	3.580(0.055)	3.308(0.045)	3.522(0.063)	
	$\boldsymbol{\varepsilon_{t-1}^2}$	0.409(0.000)	0.322(0.004)	0.371(0.001)	0.324(0.002)	0.368(0.001)	
	$\mathbf{I_{t-1}^2}$	0.283(0.012)	0.230(0.105)	0.243(0.074)	0.230(0.094)	0.239(0.089)	
	$\boldsymbol{\sigma_{t-1}^2}$	0.555(0.000)	0.625(0.000)	0.589(0.000)	0.619(0.000)	0.595(0.000)	
	AIC	7.103	7.076	7.090	7.072	7.087	
VOLATILITY	$\mathbf{c(4)\varepsilon_{t-1}^2 + c(5)\sigma_{t-1}^2}$	0.964	0.947	0.96	0.943	0.963	
	C	-0.233(0.418)	-0.059(0.837)	-0.103(0.723)	-0.073(0.798)	-0.083(0.770)	
PGARCH (1,2,1)	$\boldsymbol{\varepsilon_{t-1}}$	0.196(0.000)	0.196(0.000)	0.200(0.000)	0.196(0.000)	0.202(0.000)	
	C	3.952(0.025)	3.257(0.050)	3.601(0.055)	3.328(0.044)	3.543(0.063)	
	$\boldsymbol{\varepsilon_{t-1}^2}$	0.541(0.000)	0.430(0.000)	0.485(0.000)	0.432(0.000)	0.480(0.000)	
	$\mathbf{I_{t-1}^2}$	0.131(0.019)	0.134(0.127)	0.125(0.091)	0.133(0.114)	0.124(0.112)	
	$\boldsymbol{\sigma_{t-1}^2}$	0.554(0.000)	0.625(0.000)	0.588(0.000)	0.619(0.000)	0.594(0.000)	
	AIC	7.103	7.076	7.090	7.072	7.087	
	$\mathbf{c(4)\varepsilon_{t-1}^2 + c(5)\sigma_{t-1}^2}$	1.095	1.055	1.073	1.051	1.074	

This finding aligns with Deebom [15], who investigated Granger's causality analysis of the interaction between global macroeconomic variables and commodity price movements in Nigeria (Table 5).

Table 5: Results of the asymmetric GARCH model for returns on crude oil price (RCOP) continuations

Model	Parameter	NED	STD	GED	STDt (df = 10) ED	GED (df =1.5)	MIN AIC
	C	0.312(0.000)	0.148(0.480)	0.128(0.836)	0.166(0.460)	0.155	
	RCOP(-1)	0.204(0.000)	0.185(0.000)	0.192(0.000)	0.186(0.000)	0.194	
	Variance Equation						
	C(3)	140.630(0.081)	290.950(0.832)	318.036(0.836)	237.867(0.781)	6039.582(0.940)	
CGARCH (1,1)	C(4)	0.905(0.000)	0.994(0.000)	0.994(0.000)	0.994(0.000)	1.000(0.000)	
	C(5)	0.431(0.003)	-0.144(0.946)	-0.140(0.941)	-0.0969(0.933)	0.025(0.861)	
	C(6)	-0.009(0.948)	0.624(0.767)	0.699(0.711)	0.583(0.614)	0.531(0.000)	
	C(7)	0.080(0.241)	-0.010(0.871)	-0.010(0.856)	-0.011(0.819)	-0.008(0.794)	
	C(8)	0.936(0.000)	0.355(0.862)	0.276(0.879)	0.386(0.721)	0.438(0.0142)	
	AIC	7.125	7.071	7.082	7.067	7.079	
	$\mathbf{c(4)\varepsilon_{t-1}^2 + c(5)\sigma_{t-1}^2}$	0.927	0.979	0.975	0.969	0.969	

This section presents the results from estimating the returns of crude oil prices using both symmetric and asymmetric GARCH models (Table 6).

Table 6: Comparison of the different symmetric and asymmetric GARCH models

ARCH (1)	Normal	Student'-t	GED	SWf (df=10)	GED (df=1.5)	Min AIC	Remarks	Max Impact	Min Impact
AIC	7.173	7.138	7.149	7.135	7.145	7.135			

ARCH (2)									
AIC.	7.130	7.129	7.138	7.117	7.132	7.117			
GARCH									
AIC	7.133	7.104	7.092	7.073	7.088	7.073			
Impact of volatility	3.333	3.774	1.099	1.07	1.099			3.774	1.070
GARCH-Mean									
AIC	7.101	7.07	7.086	7.066	7.082	7.066			
Impact of Volatility	1.140	1.083	1.11	1.079	1.11			1.140	1.079
EGARCH									
AIC	7.0634	7.041	7.056	7.037	7.054	7.037	EGARCH (SWf(df=10)		
Impact of Volatility	1.573	1.496	1.538	1.496	1.528			1.573	1.496
TGARCH									
AIC	7.103	7.076	7.09	7.072	7.087	7.072			
Impact of Volatility	0.964	0.947	0.96	0.943	0.963			0.964	0.943
PARCH-Mean									
AIC	7.103	7.076	7.09	7.072	7.087	7.072			
Impact of Volatility	1.095	1.055	1.073	1.051	1.074			1.095	1.051
CARCH									
AIC	7.125	7.071	7.082	7.067	7.079	7.067			
Impact of Volatility	0.927	0.979	0.975	0.969	0.969			0.979	0.927

Initially, the standard GARCH model's orders (p, q) are identified. Then, various GARCH variants are evaluated, leading to the selection of the best-fitting GARCH(p,q) model.

Table 7: Model diagnostic check for EGARCH

EGARCH (1,1) Students with Fixed Parameter Degree of Freedom (df=10)				
	F-statistic	Prob. F(1,484)	Obs*R-squared	Prob. Chi-Square(1)
ARCH –LM	0.155	0.6938	0.156	0.693
Normality Test	skewness	Kurtosis	Jarque-Bera	P-value
	-0.528	4.652	77.960	0.000

For the selected model, several assumptions about error distributions are explored, and the related parameter estimates are presented in Tables 3, 4, and 5 (Figure 3).

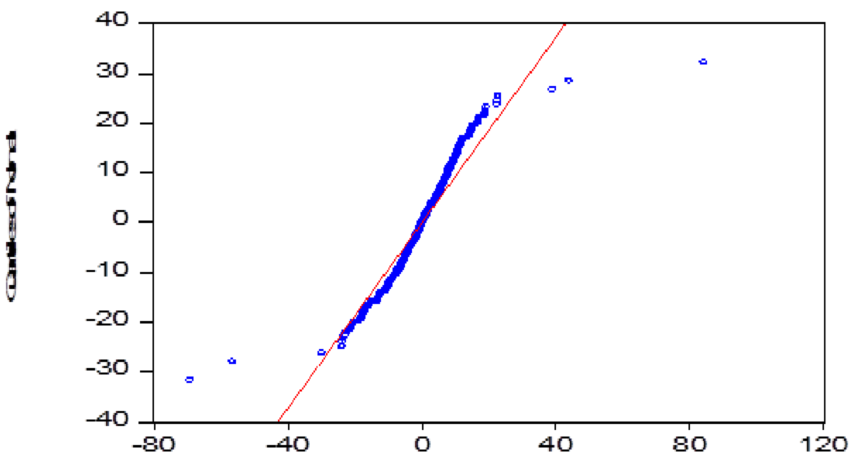

Figure 3: Q-Q plots for EGARCH (1,1) students'-t with fixed parameter degree of freedom (df=10)

The model diagnostics check carried out in this study includes: comparison of the different symmetric and asymmetric GARCH models, ARCH–LM, Heteroskedastic test, normality, Q-Qplots, and correlogram of the residuals (Figure 4).

Date: 12/24/23 Time: 05:55
Sample: 1982M01 2022M09
Included observations: 487
Q-statistic probabilities adjusted for 1 dynamic regressor

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.036	0.036	0.6343	0.426
		2	-0.051	-0.053	1.9243	0.382
		3	0.018	0.022	2.0915	0.554
		4	-0.053	-0.058	3.4918	0.479
		5	-0.025	-0.018	3.7935	0.580
		6	-0.060	-0.065	5.5492	0.476
		7	0.019	0.024	5.7201	0.573
		8	-0.036	-0.048	6.3668	0.606
		9	-0.013	-0.007	6.4543	0.694
		10	0.049	0.037	7.6686	0.661
		11	0.042	0.040	8.5413	0.664
		12	0.018	0.013	8.7101	0.727
		13	-0.043	-0.042	9.6465	0.723
		14	-0.041	-0.039	10.487	0.726
		15	-0.081	-0.079	13.811	0.540
		16	-0.022	-0.011	14.048	0.595
		17	0.041	0.034	14.916	0.602
		18	-0.028	-0.032	15.324	0.640
		19	-0.001	-0.006	15.325	0.702
		20	0.083	0.073	18.859	0.531
		21	0.026	0.011	19.202	0.572
		22	0.007	0.007	19.228	0.631
		23	-0.062	-0.068	21.229	0.567
		24	-0.053	-0.044	22.647	0.541
		25	0.035	0.050	23.265	0.562
		26	-0.071	-0.062	25.875	0.470
		27	-0.079	-0.083	29.134	0.354
		28	0.015	0.002	29.244	0.400
		29	-0.065	-0.082	31.474	0.343
		30	0.084	0.083	35.170	0.237
		31	-0.012	-0.048	35.251	0.274
		32	-0.036	-0.044	35.939	0.289
		33	0.029	0.022	36.381	0.314
		34	-0.014	0.000	36.478	0.354
		35	-0.002	0.004	36.479	0.400
		36	0.004	0.010	36.487	0.446

*Probabilities may not be valid for this equation specification.

Figure 4: Correlogram of standardised residuals for EGARCH (1,1) students'-t ' t with fixed parameter degree of freedom (df=10)

5. Discussion

Table 2 displays the findings from Engle's ARCH test alongside the Ljung-Box-Pierce Q-test regarding crude oil price returns. Both tests took place before the models were estimated. The ARCH test reveals heteroskedasticity, as the p-values are less than 0.05 for lags 1, 5, and 10. Consequently, we reject the null hypothesis, indicating the presence of ARCH effects at the 5% significance threshold. Likewise, the Ljung-Box-Pierce Q-test yields low p-values (0.000), indicating values below 0.05, except for lag 10, where the p-value exceeds 0.05. Thus, we also dismiss the null hypothesis asserting no serial correlation in the return series, aligning with the findings of Okoye et al. [3] on the empirical analysis of exchange rates, interest rates, and Nigerian agricultural export prices using VAR and variance decomposition innovation.

The estimation results are presented in Tables 3, 4, and 5. Table 3 provides the findings for the symmetric GARCH model concerning crude oil price returns, while Tables 4 and 5 present results for the asymmetric GARCH model based on five different error distributions: normal, Student's t, generalised, Student's t with fixed degrees of freedom (10), and generalised with fixed degrees of freedom. The results from the ARCH(1) model indicate statistical significance at the 5% level for all parameters, except for the ARCH(2) parameter, which is insignificant due to the presence of the second lag. According to the Akaike Information Criterion (AIC), the ARCH (1) model that uses the Student's distribution is identified as the most appropriate. In the symmetric GARCH (1,1) model, both ARCH and GARCH components in the mean and variance equations are significant at the 5% level.

Additionally, the findings reveal that the Student t distribution has the most substantial volatility effect (377.4%), followed by the normal distribution (333.3%), the generalised distribution (109.9%), the generalised distribution with fixed parameters (109.9%), and the Student s-t distribution with fixed degrees of freedom (107.9%). These outcomes are consistent with the research conducted by Damian-Effiom et al. [1], which explored the use of GARCH models to analyse returns from an all-share index in an emerging capital market. In their evaluation, the best parsimonious model, according to a standard criterion, was the GARCH in mean model utilising a Student's t error distribution with a fixed parameter (V = 5). For the asymmetric elements, the GARCH model with Student's t error distribution was preferred.

Furthermore, GARCH-in-Mean models (squared GARCH) indicate that most parameters are significant at the 1% level, apart from the normal error distribution. The findings demonstrate volatility impacts of 114% for the normal distribution, 111% for generalised (and generalised with fixed parameter), 108.3% for Student's t, and 107.9% for Student's t with fixed degrees of freedom. The asymmetric GARCH results in Tables 4 and 5 indicate that the ARCH components of the mean equation are significant at the 5% level. Likewise, the ARCH and GARCH components of the variance equations are significant. Leverage effects in EGARCH, TGARCH, PGARCH (1,2,1), and CGARCH are also significant at 5%.

This suggests that negative crude oil returns (shocks) increase volatility more strongly than positive shocks, which tend to reduce volatility. Specifically, EGARCH results show volatility impacts of 1.573% (normal), 1.496% (Student's t), 1.5376% (generalised), 1.528% (generalised with fixed parameter), and 1.496% (Student's t fixed df). TGARCH results indicate impacts of 96.4% (normal), 96.3% (generalised with fixed parameter), 96% (generalised), 94.7% (Student's t), and 94.3% (Student's t fixed df). PGARCH results show 109.5% (normal), 107.4% (Student's t fixed df), 107.3% (generalised), 105.5% (Student's t), and 105.1% (Generalised fixed parameter). CGARCH results reveal 97.9% (Student's t fixed df), 97.5% (generalised), 96.9% (Student's t), 96.9% (generalised fixed parameter), and 92.7% (normal). Model comparison (Table 6) indicates that EGARCH with Student's t distribution (fixed df = 10) yields the lowest AIC, making it the best-fitting model.

The model with the highest volatility persistence is the GARCH model with a Student's t distribution (377.4%). In contrast, the lowest persistence is observed in the CGARCH model with a Student's t distribution. The diagnostic checks (Table 7) for EGARCH yield F-statistics of 0.155 with p-values of 0.6938 at lags 1 and 2, confirming the absence of a remaining ARCH effect in crude oil price returns, even at the 1% significance level. The Jarque-Bera test confirms normality under the EGARCH-Student's t (fixed df = 10) assumption. The Q-Q plots (Figure 3) also indicate a good fit, while the correlogram of standardised residuals (Figure 4) confirms model adequacy, as the ACF and PACF probabilities exceed 5%. These outcomes are consistent with the research conducted by Deebom and Tuaneh [13] and Damian-Effiom et al. [1].

6. Conclusion

The research examined the performance of symmetric and asymmetric GARCH models in Nigeria's crude oil market from 1987 to 2023. The findings revealed an ARCH effect in the monthly returns of Nigeria's crude oil prices. The model that best captured the volatility in these prices from January 1987 to April 2023 was the General Autoregressive Conditional Heteroskedasticity (GARCH) model with a Student's t error distribution, showing a volatility impact of 377.4%. This was followed by the model using a normal error distribution, which had an impact of 333.3%. The analysis of leverage effects and news impact indicated that negative returns tend to lead to lower equity prices. This could result in a significantly increased debt-to-equity ratio. Furthermore, it was found that positive shocks have a greater effect in reducing the volatility of crude oil price returns compared to negative shock series. Among the models tested, the EGARCH with a Student's t distribution and a fixed parameter of ten proved to be the top performer, as determined by the Akaike information criterion for model evaluation. Diagnostic tests confirmed the adequacy of the EGARCH model in representing Nigeria's crude oil pricing. This suggests that shocks related to Nigeria's crude oil market tend to have lasting effects. Additionally, the long memory hypothesis model for the Nigerian crude oil prices is robust, indicating a trend that reflects an unstable market environment.

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